

Freeport AP Statistics
Chapter 9: Testing a Claim
9.1 Significance Tests: The Basis

OBJECTIVE(S):

- Students will learn how to state correct hypotheses for a significance test about a population proportion or mean.
- Students will learn how to interpret P -values in context.
- Students will learn how to interpret a Type I error and a Type II error in context, and give the consequences of each.
- Students will understand the relationship between the significance level of a test, P (Type II error), and power.

Significance Test -

1. ACTIVITY: I'm a _____ free-throw shooter!

Name _____

Free-Throw % (p) _____ NULL HYPOTHESIS (H_o)

n _____

made _____

\hat{p} _____

a. What's your opinion regarding the Null Hypotheses?

b. What affect does taking more shots have on your conclusion of the Null Hypotheses?

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- c. When is it easier to reject the Null Hypotheses regarding free-throw percentages?
2. Heading into the mayoral election, Jack is feeling fairly confident that he will be elected by obtaining more than 50% of the vote. Suppose that a random sample of 100 voters shows that 56 will vote for Jack. Is his confidence warranted? Maybe. There are actually two possible explanations for why the majority of voters in the sample seem to favor Jack.

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Null Hypotheses H_o -

Alternative Hypotheses H_a -

3. What is the difference between a one-sided alternative hypothesis and two-sided alternative hypothesis?
4. Mike is an avid golfer who would like to improve his play. A friend suggests getting new clubs and lets Mike try out his 7-iron. Based on years of experience, Mike has established that he mean distance that balls travel when hit with his old 7-iron is $\mu = 175$ yards with a standard deviation of $\sigma = 15$ yards. he is hoping that this new club will make his shots with a 7-iron more consistent (less variable), so he goes to the driving range and hits 50 shots with the new 7-iron.
 - a. Describe the parameter of interest in this setting.

- b. State appropriate hypotheses for performing a significance test.

NOTE: To recognize that a problem is calling for a two-sided test, look for key phrases such as “Has the true value **changed**?” or “Is there convincing evidence that the true value is **different**?” Also, a question might ask students to “test the claim that $\mu = 10$ ” without suggesting a direction for the alternative. In cases like this, it is appropriate to use a two-sided test.

On the AP exam, many students incorrectly use sample statistics such as _____ or _____ when stating their hypotheses. We already know the value of the sample statistic – it is the population parameter that we are trying to make an inference about.

***P*-value –**

- c. When Mike was testing a new 7-iron, the hypotheses were

$$H_o : \sigma = 15$$

$$H_a : \sigma < 15$$

where σ = the true standard deviation of the distances over which Mike hits golf balls using the new 7-iron. Based on 50 shots with the new 7-iron, the standard deviation was $s_x = 10.9$ yards. A significance test using the sample data produced a *P*-value of 0.002.

- i. Interpret the *P*-value in this context.
- ii. Do the data provide convincing evidence against the null hypothesis? Explain.

5. Was OJ Simpson innocent?
 - a. What were the two hypotheses for the OJ trial?

b. Based on the sample of evidence provided, what was the verdict?

Statistically Significant –

$P\text{-value} < \alpha \rightarrow$ \rightarrow

$P\text{-value} \geq \alpha \rightarrow$ \rightarrow

6. For his second semester project in AP Statistics, Zenon decided to investigate whether students at his school prefer name-branded potato chips to generic potato chips. He randomly selected 50 students and had each student try both types of chips, in random order. Overall, 34 of the 50 students preferred the name-brand chips. Zenon performed a significance test using the hypotheses

$$H_o : p = 0.5$$

$$H_a : p > 0.5$$

where p = the true proportion of students at his school who prefer name-brand chips. The resulting P -value was 0.0055. What conclusion would you make at each of the following significance levels?

a. $\alpha = 0.01$

b. $\alpha = 0.001$

Type I Error –

Type II Error –

- fail “**II**” reject the null hypothesis when they really should.

**Truth about
the population**

7. What type of error was committed in the OJ trial (according to popular opinion)?
8. Asked to explain the meaning of “statistically significant at the $\alpha = 0.05$ level,” a student says, “This means that the probability that the null hypothesis is true is less than 0.05.” Is this explanation correct? Why or why not?

9. The manager of a fast-food restaurant wants to reduce the proportion of drive-through customers who have to wait more than two minutes to receive their food once their order is placed. Based on store records, the proportion of customers who had to wait at least two minutes was $p = 0.63$. To reduce this proportion, the manager assigns an additional employee to assist with drive-through orders. During the next month, the manager will collect a random sample of drive-through times and test the following hypotheses:

$$H_o : p = 0.63$$

$$H_a : p < 0.63$$

where p = the true proportion of drive-through customers who have to wait more than two minutes after their order is placed to receive their food.

- a. Describe a Type I and Type II error in this setting and explain the consequences of each. Which one is more serious?
- b. Suppose that the manager decided to carry out this test using a random sample of 250 orders and a significance level of $\alpha = 0.10$. What is the probability of making a Type I error?

NOTE: Type I and Type II errors are not mistakes made by a person conducting a test. They are ultimately caused by the variability in the data collection process. Also, the probability of a Type I error and the probability of a Type II error are **INVERSELY** related.

Significance and Type I Error

10. Why don't we use a very small significance level to reduce the probability of making a Type I error?

Freeport AP Statistics
Chapter 9: Testing a Claim
9.2 Tests about a Population Proportion

OBJECTIVE(S):

- Students will learn how to check conditions for carrying out a test about a population proportion.
- Students will learn how to conduct a significance test about a population proportion if the conditions are met.
- Students will learn how to use a confidence interval to draw a conclusion for a two-sided test about a population proportion.

Power –

Power and Type II Error

11. What are three ways to increase the Power?

Power =

- a.
- b.
- c.

13. Check that the conditions for carrying out a one-sample z test for the population proportion p are met. A Gallup Poll report on a national survey of 1028 teenagers revealed that 72% of teens said they rarely or never argue with their friends. Yvonne wonders whether this national result would be true in her large high school, so she surveys a random sample of 150 students.
14. Explain why we aren't safe carrying out a one-sample z test for the population proportion p . A college president says, "99% of the alumni support my firing of Coach Boggs." You contact an SRS of 200 of the college's 15,000 living alumni to test the hypothesis $H_o : p = 0.99$.

Test Statistic –

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15. A Gallup Poll report on a national survey of 1028 teenagers revealed that 72% of teens said they rarely or never argue with their friends. Yvonne wonders whether this national result would be true in her large high school, so she surveys a random sample of 150 students. For Yvonne's survey, 96 students in the sample said they seldom or never argue with friends.
- Calculate the test statistic.
 - Find the P -value. Show this result as an area under a standard Normal curve.

One-Sample z Test for a Proportion

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When performing inference tests, beware of the **PHANTOMS!!!!**

P

H

A

N

T

O

M

S

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Test of statistic

Obtain a *P*-value.

P-value =

Make a decision about null

State your conclusion (H_a in context of the problem)

17. When the accounting firm Dewey, Cheatem and Howe audits a company's financial records for fraud, they often use a test based on Benford's law. Benford's law states that the distribution of first digits in many real-life sources of data is not uniform. In fact, when there is no fraud, about 30.1% of the numbers in financial records begin with the digit 1. However, if the proportion of first digits that are 1 is significantly different from 0.301 in a random sample of records, Dewey, Cheatem and Howe does a much more thorough investigation of the company. Suppose that a random sample of 300 expenses from a company's financial records results in only 68 expenses that begin with the digit 1.
- a. Should Dewey, Cheatem and Howe do a more thorough investigation of this company?

Parameter of interest

Hypothesis

$H_0 :$

$H_a :$

Are the conditions met?

- **Random:**

- **Normal:**

- **Independent:**

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Name of the test

Test of statistic

Obtain a P -value.

P -value =

Make a decision about null

State your conclusion (H_a in context of the problem)

DAY 3

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- b. Find and interpret a confidence interval for the true proportion of expenses that begin with the digit 1 for the company.

Parameter of interest

Are the conditions met?

- **Random:**

- **Normal:**

- **Independent:**

Name of the interval

Interval

Conclusion

- c. Use the interval from part b. to decide whether this company should be investigated for fraud.

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18. In a recent year, 73% of first-year college students responding to a national survey identified “being very well-off financially” as an important personal goal. A state university finds that 132 of an SRS of 200 of its first-year students say that this goal is important. Is there good evidence that the proportion of all first-year students at this university who think being very well-off is important differs from the national value, 73%?
- Carry out a test at the $\alpha = 0.05$ significance level to help answer this question.

Parameter of interest

Hypothesis

$H_0 :$

$H_a :$

Are the conditions met?

- **Random:**

- **Normal:**

- **Independent:**

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Name of the test

Test of statistic

Obtain a P-value.

P-value =

Make a decision about null

State your conclusion (H_a in context of the problem)

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21. Find the P -value for a test of

a. $H_o : \mu = 10$ versus $H_a : \mu > 10$ that uses a sample of size 75 and has a test statistic of $t = 2.33$.

b. $H_o : \mu = 10$ versus $H_a : \mu \neq 10$ that uses a sample of size 10 and has a test statistic of $t = -0.51$.

22. What do you do if the df you need isn't provided in Table B?

One-Sample t Test

23. Every road has one at some point – construction zones that have much lower speed limits. To see if drivers obey these lower speed limits, a police officer used a radar gun to measure the speed (in miles per hour, or mph) of a random sample of 10 drivers in a 25 mph construction zone. Here are the results:

27	33	32	21	30	30	29	25	27	34
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- a. Can we conclude that the average speed of drivers in this construction zone is greater than the posted 25 mph speed limit?

Parameter of interest

Hypothesis

$H_0 :$

$H_a :$

Are the conditions met?

- **Random:**

- **Normal:**

- **Independent:**

Name of the test

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Test of statistic

Obtain a P -value.

P -value =

Make a decision about null

State your conclusion (H_a in context of the problem)

- b. Given your conclusion in part a. which kind of mistake – a Type I or a Type II error – could you have made? Explain what this mistake means in this context.

- **Independent:**

Name of the test

Test of statistic

Obtain a P -value.

P -value =

Make a decision about null

State your conclusion (H_a in context of the problem)

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- c. Here is the Fathom output for a 95% confidence interval for the true mean width of plastic ice cubes produced this hour:

Count = 50 Mean = 29.4846 Std Dev = 0.0900274

Std Error = 0.0127318 Conf. Interval: 95.0%

Estimate: 29.4846 +/- 0.0255855 mm

Range: 29.459 to 29.5102 mm

- i. Interpret the confidence interval. Would you make the same conclusion with the confidence interval as you did with the significance test in part b.

- ii. Interpret the confidence level.

Paired Data –

Paired t Procedures –

25. For their second semester project in AP Statistics, Libby and Kathryn decided to investigate which line was faster in the supermarket: the express lane or the regular lane. To collect their data, they randomly selected 15 times during a week, went to the same store, and bought the same item. However, one of them used the express lane and the other used a regular lane. To decide which lane each of them would use, they flipped a coin. If it was heads, Libby used the express lane and Kathryn used the regular lane. If it was tails, Libby used the regular lane and Kathryn used the express lane. They entered their randomly assigned lanes at the same time, and each recorded the time in seconds it took them to complete the transaction. Carry out a test to see if there is convincing evidence that the express lane is faster.

Time in express lane (seconds)	Time in regular lane (seconds)
337	342
226	472
502	456
408	529
151	181
284	339
150	229
357	263
349	332
257	352
321	341
383	397
565	694
363	324
85	127

Parameter of interest

Hypothesis

$H_0 :$

$H_a :$

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Obtain a P -value.

P -value =

Make a decision about null

State your conclusion (H_a in context of the problem)

26. What's the difference between Statistical Significance and Practical Importance?

27. Suppose that a large school district implemented a new math curriculum for the current school year. To see if the new curriculum is effective, the district randomly selects 500 students and compares their scores on a standardized test after the curriculum change with their scores on the same test before the curriculum change. The mean improvement was $\bar{x}_d = 0.9$ with a standard deviation of $s_d = 12.0$.
- Calculate the test statistic and P -value for a test of $H_o : \mu_d = 0$ versus $H_a : \mu_d > 0$.
 - Are the results significant at the 5% level?
 - Can we conclude that the new curriculum is the cause of the apparent increase in scores?

28. Suppose you wanted to know the average GPA for students at your school who are enrolled in AP Statistics. Since this isn't a large population, you conduct a census and record the GPA for each student. Is it appropriate to construct a one-sample t interval for the population mean GPA?

29. Suppose that 20 significance tests were conducted and in each case the null hypothesis was true. What is the probability that we avoid a Type I error in all 20 tests? If we are using a 5% significance level, each individual test has a 0.95 probability of avoiding a Type I error.

**Chapter 9 Summary
INFERENCE TESTING**

	p	μ	<u>Matched Pairs</u>
Parameter of interest			
Hypothesis			
H_0 :			
H_a :			
Are the conditions met?			
• Random:			
• Normal:			
• Independent:			

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Name of the test

Test of statistic

Obtain a P -value.

P -value =

Make a decision about null

State your conclusion
(H_a in context of the problem)

