Freeport AP Statistics

Chapter 5: Probability: What Are the Chances?
5.1 Randomness, Probability, and Simulation

Chapter 5 Big Ideas:

- Probability is the basis for confidence intervals and significance tests we will encounter in chapters 8-12.
- Law of Large Numbers
- Streaks of the same outcome are often viewed with suspicion…even though they occur often just by chance.
- Simulations imitate chance processes and estimate probabilities
- Using two-way tables, tree diagrams, and Venn diagrams
- Conditional probability and independence are used throughout the rest of this course.
- Key formulas: general multiplication rule; multiplication rule for independent events; and conditional probability formula.

5.1 OBJECTIVE(S):

- ***Students will learn the idea of probability.***
- ***Students will learn the myths about randomness.***
- ***Students will learn how to simulate probability.***

Read the "Case Study" on page 287.
Get a 6-sided die and read through and conduct the "Activity" on page 288. Each of you will conduct 4 trials.

Law of Large Numbers –

Probability -

1. What is the difference between the Law of Large Numbers and the myth of the Law of Averages? View the video at the following link.

http://tinyurl.com/mcgradyAPStats-LargeNumbers

2. Athletes are often tested for use of performance-enhancing drugs. Drug tests aren't perfect – they sometimes say that an athlete took a banned substance when that isn't the case (a "false positive"). Other times, the test concludes that the athlete is "clean" when he or she actually took a banned substance (a "false negative"). For one commonly used drug test, the probability of a false positive is 0.03.
    a. Interpret this probability as a long-run relative frequency.

    b. Which is a more serious error in this case: a false positive or a false negative? Justify your answer.

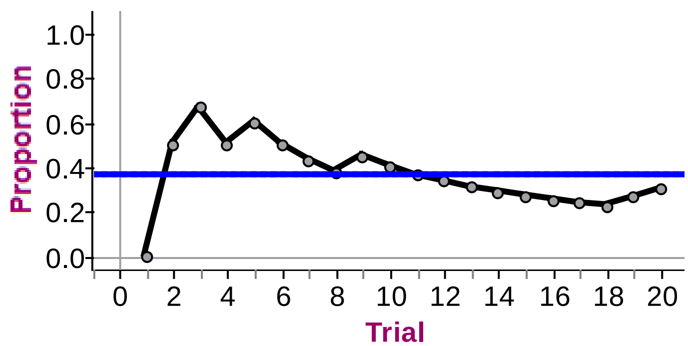
3. In the popular Texas hold 'em variety of poker, players make their best five-card poker hand by combining the two cards they are dealt with three of five cards available to all players. You read in a book on poker that if you hold a pair (two cards of the same rank) in your hand, the probability of getting four of a kind is 88/1000.
    a. Explain what this probability means.

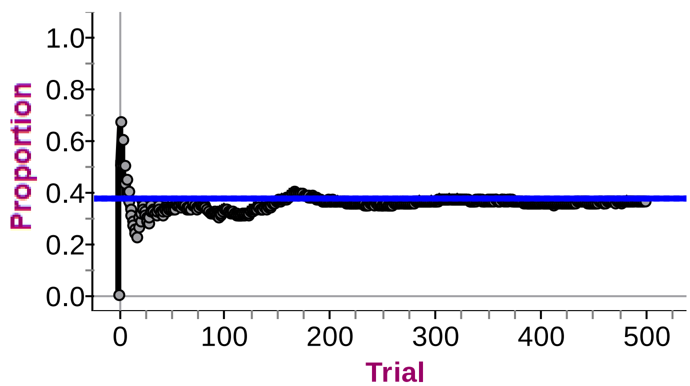
    b. Why doesn't this probability say that if you play 1000 such hands, exactly 88 will be four of a kind?

4. Suppose that four friends get together to study, and each friend brings their own textbook. When they get ready to leave the books, which were all stacked on a chair, fall to the floor. Each friend picks up a book, but does not look at the name or book number. When they get to school the next day, the students are surprised to find out that none of the four had their own book. The graphs below show the short-run and long-run behavior of the proportion of trials in which there are no matches when four students choose a book at random. The blue line is the correct probability of 0.375.

   What would you say is true after the first 20 trials?

   What would you say after 500 trials?





5. A TV weather man, predicting a colder-than-normal winter, said, “First, in looking at the past few winters, there has been a lack of really cold weather. Even though we are not supposed to use the law of averages, we are due.” Do you think that “due by the law of averages” makes sense in talking about the weather? Why or why not?

6. An uninformed gambler
   a. A gambler knows that red and black are equally likely to occur on each spin of a roulette wheel (assuming no 0 or 00). He observes five consecutive reds occur and bets heavily on black at the next spin. If it comes up red, he doubles his bet and continues this process until a black occurs. When asked why by his friends, he explains that black is "due by the law of averages." Explain to the gambler what is wrong with this reasoning.

   b. After hearing you explain why red and black are still equally likely after five reds on the roulette wheel, the gambler moves to a poker game. He is dealt five straight red cards. He remembers what you said and assumes that the next card dealt in the same hand is equally likely to be red or black. Is he right or wrong, and why?

End DAY 1

Simulation -

7. On her drive to work every day, Llana passes through an intersection with a traffic light. The light has probability 1/3 of being green when she gets to the intersection. Explain how you would use each chance device to simulate whether the light is green or not green on a given day.
    a. A six-sided die

    b. Table D of random digits

    c. A standard deck of playing cards

8. Determine whether each of the following simulation designs is valid. Justify your answer.
    a. According to a recent survey, 50% of people aged 13 and older in the United States are addicted to email. To simulate choosing a random sample of 20 people in this population and seeing how many of them are addicted to email, use a deck of cards. Shuffle the deck well, and then draw one card at a time. A red card means that person is addicted to email; a black card means he isn't. Continue until you have drawn 20 cards (without replacement) for the sample.

    b. A tennis player gets 95% of his second serves in play during practice (that is, the ball doesn't go out of bounds). To simulate the player hitting 5 second serves, look at pairs of digits going across a row in Table D. If the number is between 00 and 94, the serve is in; numbers between 95 and 99 indicate that the serve is out.

9. In the game of Scrabble, each player begins by drawing 7 tiles from a bag containing 100 tiles. There are 42 vowels, 56 consonants, and 2 blank tiles in the bag. Cait chooses her 7 tiles and is surprised to discover that all of them are vowels. We can use a simulation to see if this result is likely to happen by chance.
    a. State the question of interest using the language of probability.

    b. How would you use random digits to imitate one repetition of the process? What variable would use measure?

    c. Use the line of random digits below to perform one repetition. Copy these digits onto your paper. Mark directly on or above them to show how you determined the outcomes of the chance process.

       00694 05977 19664 65441 20903 62371 22725 53340

    d. In 1000 repetitions of the simulation, there were 2 times when all 7 tiles were vowels. What conclusion would you draw?

    e. About 3% of the times, the first player in Scrabble can “bingo” by playing all 7 tiles on the first turn. How many games of scrabble would you expect have to play, on average, for this to happen? Design and carry out a simulation to answer this question.

DAY 2

Freeport AP Statistics

Chapter 5: Probability: What Are the Chances?

5.2 Probability Rules

OBJECTIVE(S):

- ***Students will learn how to describe a probability model for a chance process.***
- ***Students will learn how to use basic probability rules, including the complement rule and the addition rule for mutually exclusive events.***
- ***Students will learn how to use a Venn diagram to model a chance process involving two events.***
- ***Students will learn how to use the general addition rule to calculate*** $P(A \cup B)$**.**

Sample Space –

Probability Model -

Event –

Mutually Exclusive (Disjoint) Events -

State the Basic Probability Rules:

-
-
-

-
-

General Addition Rule for Two Events

10. Imagine tossing a fair coin 3 times.
    a. What is the sample space for this chance process?

    b. What is the assignment of probabilities to outcomes in this sample space?

    c. Define event *B*: get more heads than tails. Find $P(B)$.

11. View the probability models for rolling a die on page 315, #44. Answer the question. Explain.

    Model 1:

    Model 2:

    Model 3:

    Model 4:

12. Canada has two official languages, English and French. Choose a Canadian at random and ask, "What is your mother tongue?" Here is the distribution of responses, combining many separate languages from the broad Asia/Pacific region:

Language:	**English**	**French**	**Asian/Pacific**	**Other**
Probability:	0.63	0.22	0.06	???

    a. What probability should replace "???" in the distribution? Why?

    b. What is the probability that a Canadian's mother tongue is not English?

c. What is the probability that a Canadian's mother tongue is a language other than English or French?

DAY 3

VENN DIAGRAMS AND PROBABILITY

13. A company that offers courses to prepare students for the Graduate Management Admissions Test (GMAT) has the following information about its customers: 20% are currently undergraduate students in business; 15% are undergraduate students in other fields of study; 60% are college graduates who are currently employed; and 5% are college graduates who are not employed. Choose a customer at random.
    a. What's the probability that the customer is currently an undergraduate? Which rule of probability did you use to find the answer?

    b. What's the probability that the customer is not an undergraduate business student? Which rule of probability did you use to find the answer?

14. The two-way table below describes the members of the U.S. Senate in a recent year.

	Male	**Females**
Democrats	47	13
Republicans	36	4

a. Who are the individuals? What variables are being measured?

b. If we select a U.S. senator at random, what's the probability that we choose:

- a Democrat?
- a female?
- a female Democrat?
- a female or a Democrat?

c. Construct a Venn diagram that models the chance process using events *R*: is a Republican, and *F*: is female.

d. Find $P\left(R \cup F\right)$. Interpret this value in context.

e. Find $P\left(R^c \cap F^c\right)$. Interpret this value in context.

15. Shuffle a standard deck of playing cards and deal one card. Define events *J*: getting a jack, and *R*: getting a red card.
    a. Construct a two-way table that describes the sample space in terms of events *J* and *R*.

    b. Find $P(J)$ and $P(R)$.

    c. Describe the "*J* and *R*" in words. Then find $P(J and R)$.

    d. Explain why $P(J or R) \neq P(J)+P(R)$. Then use the general addition rule to compute $P(J or R)$.

16. A recent census at a major university revealed that 40% of its students primarily used Macintosh computers (Macs). The rest mainly used PCs. At the time of the census, 67% of the school's students were undergraduates. The rest were graduate students. In the census, 23% of respondents were graduate students who said that they used PCs as their main computers. Suppose we select a student at random from among those who were part of the census.
    a. Assuming that there were 10,000 students in the census, make a two-way table for this chance process.

    b. Construct a Venn diagram to represent this setting.

c. Consider the event that the randomly selected student is a graduate student who uses a Mac. Write this event in symbolic form based on your Venn diagram in part b.

d. Find the probability of the event described in part c. Explain your method.

DAY 4

Freeport AP Statistics

Chapter 5: Probability: What Are the Chances?

5.3 Conditional Probability and Independence

OBJECTIVE(S):

- ***Students will learn how to use a tree diagram to describe chance behavior.***
- ***Students will learn how to use the general multiplication rule to solve probability questions.***
- ***Students will learn how to compute conditional probabilities.***
- ***Students will learn how to determine whether two events are independent.***
- ***Students will learn how to find the probability that an event occurs using a two-way table.***
- ***Students will learn how to use the multiplication rule for independent events to compute probabilities.***

Conditional Probability -

Independent Events -

General Multiplication Rule -

17. In 1912 the luxury liner Titanic, on its first voyage across the Atlantic, struck an iceberg and sank. Some passengers got off the ship in lifeboats, but many died. The two-way table gives information about adult passengers who lived and who died, by class of travel. Suppose we choose an adult passenger at random.

	Survival Status	
Class of Travel	**Survived**	**Died**
First Class	197	122
Second Class	94	167
Third Class	151	476

a. Given that the person selected was in first class, what's the probability that he or she survived?

b. If the person selected survived, what's the probability that he or she was a third-class passenger?

c. Find $P(survived \mid \sec ond\ class)$.

d. Find $P(survived)$.

e. Determine whether the events "survived" and "second class" are independent. Explain your reasoning.

18. The two-way table describes the 595 students who responded to a school survey about eating breakfast. Suppose we select a student at random. Consider events B: eats breakfast regularly, and M: is male.

	Male	**Female**	**Total**
Eats breakfast regularly	190	110	**300**
Doesn't eat breakfast regularly	130	165	**295**
Total	**320**	**275**	**595**

a. Find $P(B \mid M)$. Explain what this value means.

b. Find $P(M \mid B)$. Explain what this value means.

c. Are the events B and M independent? Justify your answer.

19. Here is the distribution of the adjusted gross income (in thousands of dollars) reported on individual federal income tax returns in a recent year:

Income	<25	25-49	50-99	100-499	≥ 500
Probability	0.431	0.248	0.215	0.100	0.006

a. What is the probability that a randomly chosen return shows an adjusted gross income of $50,000 or more?

b. Given that a return shows an income of at least $100,000? —

20. A shipment contains 10,000 switches. Of these, 1,000 are bad. An inspector draws 2 switches at random, one after the other.

a. Draw a tree diagram that shows the sample space of this chance process.

b. Find the probability that both switches are defective.

DAY 5

Multiplication Rule for Independent Events -

Conditional Probability Formula-

Note: Conditional probability questions can be solved using a tree diagram or a two-way table. If the problem provides conditional probabilities use a _______________________. If the problem provides counts or proportions of people in different categories, use a ____________________________.

21. 93% of teenagers are online and that 55% of online teens have posted a profile on a social-networking site. Of online teens with a profile, 76% have placed comments on

a friend's blog. What percent of all teens are online, have a profile, and comment on a friend's blog. Show your work.

22. The voters in a large city are 40% white, 40% black, and 20% Hispanic. (Hispanics may be of any race in official statistics, but here we are speaking of political blocks.) A mayoral candidate anticipates attracting 30% of the white vote, 90% of the black vote, and 50% of the Hispanic vote.

    a. What percent of the overall vote does the candidate expect to get?

b. If the candidate's predictions come true, what percent of her votes come from black voters?

23. A player serving in tennis has two chances to get a serve into play. If the first serve goes out of bounds, the player serves again. If the second serve is also out, the player loses the point. Here are probabilities based on four years of the Wimbledon Championship:

$$P(1st\ serve\ in) = 0.59 \qquad P(win\ point \mid 1st\ serve\ in) = 0.73$$

$$P(2nd\ serve\ in \mid 1st\ serve\ out) = 0.86$$

$$P(win\ point \mid 1st\ serve\ out\ and\ 2nd\ serve\ in) = 0.59$$

What is the probability that the serving player wins the point? Show your work.

24. A recent census at a major university revealed that 40% of its students mainly used Macintosh computers (Macs). The rest mainly used PCs. At the time of the census, 67% of the school's students were undergraduates. The rest were graduate students. In the census, 23% of the respondents were graduate students who said that they used PCs as their primary computers. Suppose we select a student at random from among those who were part of the census and learn that the student mainly uses a Mac. Find the probability that this person is a graduate student. Show your work.

25. Are false positives too common in some medical tests? Researchers conducted an experiment involving 250 patients with a medical condition and 750 other patients who did not have the medical condition. The medical technicians who were reading the test results were unaware that they were subjects in an experiment.
    a. Technicians correctly identified 240 of the 250 patients with the condition. They also identified 50 of the healthy patients as having the condition. What were the false positive and false negative rates for the test?

    b. Given that a patient got a positive test result, what is the probability that the patient actually had the medical condition? Show your work.

CHAPTER 5

DAY 6