

Part 1: Multiple Choice. Circle the letter corresponding to the best answer.

1. A marketing survey compiled data on the total number of televisions in households. If X = the number of televisions in a randomly-selected household, and we omit the rare cases of more than 5 televisions, then X has the following distribution:

X	0	1	2	3	4	5
$P(X)$	0.24	0.37	0.20	0.11	0.05	0.03

What is the probability that a randomly chosen household has at least two televisions?

- (a) 0.19 (b) 0.20 (c) 0.29 (d) 0.39 (e) 0.61

2. A random variable X has a probability distribution as follows:

X	0	1	2	3
$P(X)$	2k	3k	13k	2k

Where k is a positive constant. The probability $P(X < 2.0)$ is equal to

- (a) 0.90. (b) 0.25. (c) 0.65. (d) 0.15. (e) 1.00.

3. A business evaluates a proposed venture as follows. It stands to make a profit of \$10,000 with probability $3/20$, to make a profit of \$5000 with probability $9/20$, to break even with probability $5/20$, and to lose \$5000 with probability $3/20$. The expected profit in dollars is

- (a) 1500. (b) 0. (c) 3000. (d) 3250. (e) -1500.

4. Roll one 8-sided die 10 times. The probability of getting exactly 3 sevens in those 10 rolls is given by

(a) $10 \frac{1}{8}^3 \frac{7}{8}^5$

(b) $10 \frac{1}{8}^3 \frac{7}{8}^7$

(c) $8 \frac{1}{8}^3 \frac{7}{8}^5$

(d) $8 \frac{1}{8}^3 \frac{7}{8}^7$

(e) $10 \frac{7}{8}^3 \frac{1}{8}^7$

5. X and Y are independent random variables, and a and b are constants. Which one of the following statements is true?

(a) $\sigma_{X+Y} = \sigma_X + \sigma_Y$

(b) $\text{Var}(X - Y) = \text{Var}(X) + \text{Var}(Y)$

(c) $\text{Var}(a + bX) = b\text{Var}(X)$

(d) $\sigma_{X-Y} = \sigma_X - \sigma_Y$

(e) $\text{Var}(X+Y) = \sqrt{\text{Var}(X^2) + \text{Var}(Y^2)}$

6. Let the random variable X represent the profit made on a randomly selected day by a certain store. Assume that X is Normal with mean \$360 and standard deviation \$50. What is $P(X > \$400)$?

(a) 0.2119

(b) 0.2881

(c) 0.5319

(d) 0.7881

(e) 0.8450

7. You select 40 cards from a standard deck of 52 cards. Let Y be the number of red cards (hearts or diamonds) in the 40 cards selected. Which of the following best describes this setting?

(a) Y has a binomial distribution with $n = 40$ observations and probability of success $p = 0.5$.

(b) Y has a binomial distribution with $n = 40$ observations and probability of success $p = 0.5$, provided the deck is shuffled well.

(c) Y has a binomial distribution with $n = 40$ observations and probability of success $p = 0.5$, provided that after selecting a card it is replaced in the deck and the deck is shuffled well before the next card is selected.

(d) Y has a geometric distribution with $n = 40$ observations and probability of success $p = 0.5$.

(e) Y has a geometric distribution with $n = 52$ observations and probability of success $p = 0.5$.

8. In a large population of college students, 20% of the students have experienced feelings of math anxiety. If you take a random sample of 10 students from this population, the mean and standard deviation of the number of students in the sample who have experienced math anxiety is:

(a) $\mu = 1.6$; $\sigma = 1.414$

(b) $\mu = 1.6$; $\sigma = 1.265$

(c) $\mu = 2$; $\sigma = 1.6$

(d) $\mu = 2$; $\sigma = 1.414$

(e) $\mu = 2$; $\sigma = 1.265$

9. Which of the following is a true statement?

(a) The binomial setting requires that there are only two possible outcomes for each trial, while the geometric setting permits more than two outcomes.

(b) A geometric random variable takes on integer values from 0 to n .

(c) If X is a geometric random variable and the probability of success is 0.85, then the probability distribution of X will be skewed left, since 0.85 is closer to 1 than to 0.

(d) An important difference between binomial and geometric random variables is that there is a fixed number of trials in a binomial setting, and the number of trials varies in a geometric setting.

(e) The distribution of every binomial random variable is skewed right.

10. Jen's commute to work requires that she take the Blue subway line, then transfer to the Red line. The length of the trip on the Blue line has a mean of 18 minutes with a standard deviation of 2 minutes. The Red line trip takes 12 minutes with a standard deviation of 1 minute. The waiting time between when she gets off the Blue line and her Red line train arrives has mean of 10 minutes and a standard deviation of 5 minutes. Assume (perhaps unrealistically) that these times are independent random variables. What are the mean and standard deviation of her entire commute?
- (a) Mean = 40 minutes; Standard deviation = 8 minutes
 - (b) Mean = 40 minutes; Standard deviation = 5.48 minutes
 - (c) Mean = 40 minutes; Standard deviation = 2.83 minutes
 - (d) Mean = 30 minutes; Standard deviation = 5.48 minutes
 - (e) Mean = 30 minutes; Standard deviation = 8 minutes

Part 2: Free Response

Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

11. ACT scores for the 1,171,460 members of the 2004 high school graduating class who took the test closely followed the Normal distribution with mean 20.9 and standard deviation 4.8. Choose two students independently and at random from this group.
- (a) What is the expected difference in their scores?

 - (b) What is the standard deviation of the difference in their scores?

 - (c) Find the probability that the difference in the two students' scores is greater than 6.

12. A company that rents DVDs from vending machines in grocery stores has developed the following probability distribution for the random variable X = the number of DVDs a customer rents per visit to a machine.

X	1	2	3	4
$P(X)$	0.5	0.3	0.1	0.1

(a) Find and interpret the mean (expected value) of X .

(b) Find and interpret the standard deviation of X .

(c) Suppose the profit the company makes on each customer visit is \$0.75 per DVD minus \$0.05 “fixed costs.” That is, if P = profit, then $P = 0.75X - 0.05$. Use a linear transformation of your results in (a) and (b) to find the mean and standard deviation for P .

13. A professional soccer player succeeds in scoring a goal on 84% of his penalty kicks. Assume that the success of each kick is independent.
- (a) In a series of games, what is the probability that the first time he fails to score a goal is on his fifth penalty kick?
- (b) What is the probability that he scores on 5 or fewer of his next 10 penalty kicks?
- (c) Suppose that our soccer player is out of action with an injury for several weeks. When he returns, he only scores on 5 of his next 10 penalty kicks. Is this evidence that his success rate is now less than 84%? Explain.